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Dissertation
AN INTERFERONETER METHOD OF DFTERMINING
WAVE LENGTHS IN THE HYDROGEN SPECTRUM
by
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## UUTLINE

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## INTRODUC TION

Much theoretical work is being done on the structure of the hydrogen atom and molecule. The only way to check this theory is to measure the wave lengths of light emitted. The molecular spectrum in particular is very rich in lines in all parts of the visible spectrum as well as in the ultra violet and infra red.

If experimental work is to be used to check theory, the wave length measurements must be made to a high degree of accuracy and practically all the lines in the spectrum must be measured.

Most of these wave length measurements are made by means of a grating in the manner discussed in the writer's M.A. thesis These measurements can be made with a fair degree of speed but absolute measurements cannot be made accurately with a grating. However, accurate wave lengths may be obtained in terms of the wave lengths of other lines on the plate. This means that the wave lengths of certain lines must be known and used as standards. At the present time it is necessary to photograph the iron spectrum from a Pfund arc on the same plate as the hydrogen spectrum except for a few limited regions where

1. Lewis S. Combes M.A. Thesis. Boston University, 1928. Determination of Wave lengths of Certain Lines in the Secondary spectrum of Hydrogen.
2. R. G. Lacount and R. E. Hodgdon. Phys Rev. 52,98,1937. Interferometer Wave Lengths in the Secondary Spectrum of Hydrogen.
hydrogen standards have been accurately determined ${ }^{2}$.
The main drawback to using the iron lines as standards, aside from the extra work involved in arranging the apparatus, is that iron lines often overlap and screen hydrogen lines, which makes accurate measurement of these wave lengths impossible. Also one must take separate plates of the hydrogen and iron spectra and be very careful to distinguish between the iron and hydrogen lines on the plate being measured.

If a few hydrogen lines spaced throughout the spectrum could be measured accurately they could be used as standards and thus speed up the grating measurements considerably. It is the purpose of this paper to discuss a method by which accurate interferometer measurements can be made of certain lines in the hydrogen spectrum to serve as standards.

## HISTORICAL

All accurate interferometer measurements of wave length start with Michelson's determination of the standard meter in terms of the wave lengths of the three cadmium lines. His work is generally accepted as the most accurate yet made and his determination of the wave length of the red cadmium line is accepted for the primary standard.

Using this cadmium line as a standard several observers have measured many lines in the iron spectrum using the standard













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Pfund arc ${ }^{3}$ as the source. In 1928 the International Astronomical Union selected certain wave lengths, for which there was good agreement among the several observers, as secondary standards ${ }^{4}$.

The interferometer used in the work described in this paper was developed by Fabry and Perot. It consists of two plane parallel glass plates, $65 \%$ silvered, separated by means of a fused quartz cylinder 10 mm . long, and held in place by three spring clips. The pressure of each spring clip is controlled by an adjusting screw. The two partially silvered surfaces must be very accurately parallel and this condition may by secured by varying the pressure of the spring clips by means of the adjusting screws. This type of interferometer is called an etalon. Etalons were used by all the above mentioned investigators in their measurements of the secondary iron standards.

The first work in making interferometer measurements in the molecular spectrum of hydrogen was done by Gale, Monk, and Lee ${ }^{6}$ in which they made a few scattered measurements throughout the visible spectrum. Their probable errors are rather
3. A.H.Pfund. Astrophys. Jour. 27,296,1908
4. Transactions of International Astronomical Union.III, 86, 1928
5. C. Fabry and Perot. Astrophys. Jour. 15,73,1902
6. H.G.Gale, G.S.Monk and K.O.Iee. Astrophys. Jour. 67,89,1928




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large for reasons that will be discussed later and their results are not accurate enough to use as standards in the hydrogen spectrum.

The work of Lacount ${ }^{7}$ and Lacount and Hodgdon ${ }^{2}$ in obtaining interferometer measurements in the hydrogen spectrum was more accurate. Their probable errors being about one fourth the probable errors of Gale, Monk and Lee's ${ }^{6}$ best results. In their work the etalon used was identical with the one used in this investigation as was some of the other apparatus which will be described later.

These few investigators are the only ones to publish results of interferometer measurements in the hydrogen spectrum.

## APPARATUS

The apparatus used in this work is located in the Physics Research Room in the basement of the Boston University building at 688 Boylston Street in Boston at the corner of Exeter Street. The vibrations from subway trains under Boylston Street (and now Exeter Street) make necessary a special mounting of the apparatus to "damp out" these vibrations. Also the main steam pipes which pass overhead in the research room require special insulation around the apparatus in the form of a "house" which encloses it completely. The mounting and the insulation are
7. R.G.Jacount Ph.D. Dissertation. Boston University 1935










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Figure
Not drawn to scale
Apparatus on Vibrationless Mounting
in Insulating House
W Wall of insulating house
E Etalon
0 Opening in wall
Z Slide to close opening $T_{1,2}$ Thermometers
$\mathrm{M}^{\prime 2}$, Plane mirror. Front silvered
$D_{l, 2}$ Diaphragms
$M_{c}$ Concave mirror. Front silvered
S Adjustable slit
L, Achromatic lens
G. Grating 15000 lines per inch

H Plate holder
described in detail in an article by Kent and Lacount. ${ }^{8}$ The apparatus and optical system will be described in two parts. The part on the vibrationless mounting inside the insulating house is shown in Fig. l. 0 is an opening in the house through which light from the source enters. This opening may be closed with a slide to make the house light tight. $\mathbb{M}_{1}$ is a plane mirror used simply to change the direction of the path of light. $D_{1}$ and $D_{2}$ are diaphragms, one on each side of the etalon $E$. $M_{c}$ is a concave mirror ( $f=l$ meter) which focuses the interference fringes produced by the etalon on the slit $S$. $I_{l}$ is a 30 ft . focal length acromatic lens. $S$ is at the focus of $L_{1}$, hence light striking grating $G$ is parallel. $G$ is a reflection grating. It has 15000 lines per inch on a plane glass surface coated with aluminum. It was ruled by R. W. Wood. The licht is reflected from the grating back through lens $I_{1}$ which focuses it on the plate holder $H$ which is directly below slit $S . T_{1}$ and $T_{2}$ are thermometers.

The apparatus outside the insulating house is shown diagrammatically in Fig. 2. With the exception of the transformer V , switches $\mathrm{K}_{1}, \mathrm{~K}_{2}$ and $\mathrm{K}_{3}$ and resistance $\mathrm{R}_{1}$ everything is mounted on one table in the approximate positions shown in the diagram. The transformer is on the floor and the 30000 volt secondary line is well insulated on an overhead framework and drops directly down to the terminals of the tube $S_{1}$. The resis-
8. Norton A. Kent and Reginald G. Lacount. Jour. Optical Soc of America 28,7,1938






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Figure 2
Not drawn to scale

Vacuum System
$\mathrm{H}_{2}$ Hydrogen generator
$J_{1,2}$ Drying tubes
$F$ Storage flask
S, Discharge tube. Water
cooling system not shown
$X_{1,2}$ Mercury traps
B McLeod gauge
Diffusion pump
$P_{2} H_{y}$-vac oil pump. Not shown.
$C_{1,2,4 \% 6,6}$ Stop cooks
D Flask of deuTerium. Not used in this work.

Electrical System
$S_{2}$ Pfund arc
U Step-up transformer. 30000 v .
A Ammeter
MA Milliammeter
$R_{1,2,3}$ Resistors
$K_{1,2,3,4,5}$ Double pole switches
Optical System
$N$ Glass shield
$L_{2,3}$ Achromatic Lenses
$I^{2,3}$ Screen with horizontal slot
$M_{2,3}$ Plane mirrors. Front silvered

tance $R_{1}$ in which considerable energy is dissipated as heat is located on the wall outside the research room.

In the optical system, mirror M2 can be swung back out of the way as shown by the dotted lines, in which case light from the capillary of $S_{1}$ is focused by lens $I_{2}$ on the diaphragm $D_{1}$ (Fig. I). With $M_{2}$ in position the light entering the house through 0 comes from the arc S2. The length of the arc is 12 mm . Lens $I_{3}$ forms an image (magnification equal to 10 ) on screen I. Light from the exact center of the image passes through a horizontal slot 8 mm . wide in the screen. It is then reflected from mirrors $\mathbb{M}_{3}$ and $\mathbb{M}_{2}$ and focused by the lens $L_{2}$ on diaphragm $D_{l}$ (Fin. I). Thus the 8 mm . slot in screen I acts as the source of light when the iron arc is used. Since the magnification of the image on the screen I is lo, light from only 0.8 mm . in the center of the arc itself is used.

## PROCEDURE

Grating plates of the iron spectrum alone and the hydrogen spectrum alone must be taken in order to identify the lines on the interferometer plates. Therefore the etalon was not used at first. Lens $I_{2}$ was adjusted so that an image of the capillary tube $S_{1}$ was focused on the slit $S$ or if mirror $M_{2}$ was in position an image of the slot in screen I was focused on the slit. In order to make sure that all the optical apparatus was














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properly lined up, a screen with a hole one inch square in the center was placed just in front of lens $I_{l}$, with the hole on the optical axis of the lens. Then a lighted lamp bulb was placed just behind this hole (between the screen and lens $I_{l}$ ). Light from this source then passed through the slit $S$ (opened wide), was reflected from $M_{c}$ and $M_{I}$ and focused by $L_{2}$ on the capillary tube. Then if the light passed through the capillary tube of $S_{1}$ everything was lined up properly. Then $M_{2}$ was swung into position and adjustments made so that the image of the light came exactly at the center of the slot in screen $I$. Further adjustments of $I_{3}$ and arc $S_{2}$ were made so that the image formed by $I_{3}$ came in the center of the arc itself. These adjustments were very important. If they were not accurately made the intensity of the upper half of the lines on the plate were not the same as of the lower half. The grating was then adjusted for the region between 4800 and $5800 ~ \AA u$ in the first order spectrum. Then the grating was turned and tilted until the spectrum was parallel to and centered in the plate holder H.

The Dewar flask surrounding the mercury trap $X_{1}$ had been packed with a slush of solid carbon dioxide and acetone and the pumps had been running for many hours while the adjustments were being made. This was to remove all traces of air and water vapor from the vacuum system. Fach night some hydrogen was allowed to enter the system from generator $\mathrm{H}_{2}$ and in the
























morning the pumps were started again. This was repeated for several days. It is interesting to note here that the pressure always decreased considerably over night due undoubtedly to the absorption of the hydrogen by the platinum electrodes in the discharge tube. This proved that the vacuum system was tight.

The last step in adjusting the apparatus was to get the plate holder into the position of best focus. Using the iron arc as a source, small slant plates were taken in the center of the plate holder and the place where the lines were sharpest noted. Small vertical plates were taken at intervals of 1 mm . on each side of this position and the best focus determined in this way. In the same way the ends and intermediate portion of the plate holder were brought into focus. A grating plate of the iron arc was taken. Next with the hydrogen in the tube at a pressure of approximately 0.3 mm . of mercury a grating plate of the hydrogen spectrum was taken. Finally a plate of both hydrogen and iron was taken.

The etalon was now placed in the system at E (Fig. 1). The incandescent lamp and screen were placed in front of lens Il as in lining up the optical system before and the etalon and diaphragms $D_{1}$ and $D_{2}$ were centered with the image of this light. To make sure that the etalon was normal to the optical axis of the system, it was turned so that the light coming through the slit was reflected from the etalon back onto the
slit again. To make sure that the etalon plates were parallel, the iron arc was replaced with a mercury arc. The rings produced by the interference of light from the green line of mercury were visible on looking into the etalon. If the etalon plates were not parallel, the diameters of the rings changed as the eye was moved from side to side. The adjusting screws were turned until the ring diameters were constant as the eye was moved in any direction. Then the etalon plates were parallel. After rechecking the rest of the apparatus to make sure that all parts were centered on the optical axis, everything was ready for the exposure of an interferometer plate.

The plates used were Eastman Spectroscopic plates type I-D, emulsion \#69338. After the insulating house was made light tight by placing the slide over opening 0 , a plate was put in the plate holder at night. By morning the temperature inside the house was constant. Then the vacuum system was pumped out and fresh hydrogen let into the system until the pressure was 0.3 mrn . of mercury. Since the plates in this particular batch were not very sensitive (although very fine grained) an exposure of 12 hours was necessary. The iron arc was exposed 10 seconds at one hour intervals (130 seconds in all). The pressure was frequently checked and more hydrogen let in if it dropped below 0.3 mm . The current through the tube was $220 \mathrm{~m} . a$. Typical readings for thermometers $T_{1}$ and $T_{2}$ (for plate 14 ) were as follows. After putting in the plate, l2 hours before expo-

sure, $T_{I}=25.9^{\circ} \mathrm{C}$. and $T_{2}=25.7^{\circ} \mathrm{C}$. Here the doors of the house had been open during adjustments. At the beginning of the twelve hour exposure $T_{I}=24.91^{\circ} \mathrm{C} ., T_{2}=24.70^{\circ} \mathrm{C}$. At the end of the exposure $T_{I}=24.90^{\circ} \mathrm{C} ., T_{2}=24.68^{\circ} \mathrm{C}$. The largest change during a twelve hour exposure in the summer was $0.4^{\circ} \mathrm{C}$. (Plate 13).

The plates were all developed immediately after exposure. The development time was 6 minutes. The developer formula was the Eastman D-19.

By comparing the grating plates of the iron spectrum, the hydrogen spectrum, and the two spectra together, eleven iron lines distributed across the plate were selected to serve as iron standards. Twenty-five hydrogen lines averaging $40{ }^{\circ} \mathrm{A}$ apart were selected to be measured as tertiary standards. These lines were marked on each of the six etalon plates.

The plates were measured with a Gaertner comparator. They were placed in the comparator so that the diameters of the rings to be measured were parallel to the horizontal cross-hair in the telescope. The diameters were measured first with the long wave length end of the plate down (Red lower) and then up (Red upper). Four settings were made on each side of a ring, two with the vertical cross-hair approaching from the left, and two from the right. The method of recording these data is shown below in table 2 .
















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## CALCULATIONS

The theory of the formation of interference rings by means of an etalon was fully developed by Lacount ${ }^{77}$ in his Ph D. thesis and consequently only a brief summary will be given here.

If $t$ is the distance between the reflecting surfaces of the etalon then the path difference of the interfering rays for normal incidence will be $2 t$ and $\frac{2 t}{\lambda}=$ the order of interference for wave length $\lambda$ at nomal incidence. This will be the order of interference at the center of the ring system. Call this order of interference $p$, then

$$
\begin{equation*}
p \lambda=2 t \tag{1}
\end{equation*}
$$

Let $P=$ the integral order of interference
Let $\varepsilon=$ the fractional order of interference, then $p=P+\varepsilon$

For example, if $p=37740.182$ then $P=37740$. and $\varepsilon=.182$

It can also be shown that $n+\varepsilon=K D_{n}^{2}$
Where $n=$ number of ring $(0,1,2,3$, etc. starting from the center). Note that the first ring is numbered 0 , the second $I$, etc.
$D_{n}=$ the diameter of the $n$th ring
$K=a$ constant for any one line
If the magnification is constent for the entire plate, it



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can be shown that

$$
\begin{equation*}
K \lambda=\text { a constant for the whole plate } \tag{4}
\end{equation*}
$$

where $K=$ the above constant for a line of wave length $\lambda$. $\varepsilon$ may be calculated for any line by means of equation 3 if two or more values of $D_{n}^{2}$ are known. The readings taken on the iron standards are used to calculate $2 t$. An approximate value of $2 t$ must be known, either from a micrometer measurement or some other source. In this case an approximate value of 18.564 mm . was obtained from Lacount since he had previously used the same etalon for similar measurements. Dividing $2 t$ by $\lambda$ for one iron standard gives a tentative value of $P$ which is probably not in error by more than $\pm 1$. Then $p=P+\varepsilon$ and we can get a corrected value of $2 t$ by multiplying $\lambda$ by this value of $p(1)$. With this new value of $2 t$ we calculate tentative values of $p$ for each of the iron lines and compare the decimal part of $p$ with $\varepsilon$ in each case. We then try increasing the value of $P$ for the first iron line by one, get a new corrected value of $2 t$ and get new values of $p$ for each iron line as before. We then decrease $P$ by one and repeat. For one of these sets of values of $p$ the decimal parts should agree well with the values of $\mathcal{E}$. Then the value of $P$ which gave this set of values is correct. If none of the sets of values checks well we must continue to increase or decrease $P$ until good agreement is secured. When this result is achieved the integral part of $p$ is the correct value of $P$ for each iron line. We



Figure 3
Graph of $K D_{n}^{2}=n+E-D_{n}^{2}$ plotted against $n$

then add to this the calculated value of $\mathcal{E}$ in each case to get the correct value of $p$. Then $2 t=p \lambda$ for each line. The mean value of $2 t$ is calculated from the eleven values thus obtained.

Having found the correct value of $2 t$ we may get $P$ for each hydrogen line by dividing $2 t$ by $\lambda$. To get this, $\lambda$ must be known accurately to 0.1 Au. but measurements as accurate as this may be made from a grating plate. These measurements had already been made in this part of the spectrum by Gale, Monk and Lee ${ }^{6}$ and their values were used. Then $p=P+\varepsilon$ and $a$ corrected value of $\lambda$ is calculated from $\lambda=\frac{2 t}{p}$.

Since the accuracy of all the measurements depends upon the accuracy with which the fractional order of interference, $\varepsilon$, can be determined, various methods of calculating this quantity for each line will be discussed. It may help to give a graphical illustration of equation 3 . In figure 3 values of $D_{n}^{2}$ as ordinates are plotted against values of $n$ as abscissas When $a_{n}^{2}=0, n=-\varepsilon$. Thus $\varepsilon$ is numerically equal to the $n$ intercept of this curve which is of course a straight line.

Hence it is simply necessary to find the slope of the curve and divide it into $D_{o}^{2}$ to obtain $\varepsilon$. This was the method used by Gale, Monk and Lee in obtaining their interferometer wave lengths ${ }^{6}$. This method, however, does not give consistently accurate values of $\mathcal{\varepsilon}$. From figure 3 we see that $\varepsilon$ is obtained by extrapolation. A slight error in the slope of the




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line may cause a large error in the value of $\varepsilon$ since swinging the line slightly will make a relatively large difference in the point of intersection with the $n$ axis. This is shown very clearly if one tries to obtain $\varepsilon$ by a graphical method. Values of $D_{n}^{2}$ may be plotted against $n$ as in figure 3 and then a straight line obtained by holding a tightly stretched thread so that it best fits the plotted points. Careful placing of the thread several times will give as many values of $\varepsilon$ which will be found to vary considerably.

The above method does not make use of equation 4 ,
$K \lambda=$ the plate constant
If we use that equation we can obtain $K \lambda$ for each line on the plate and get a mean plate constant for the whole plate. If then this mean plate constant be divided by each individual wave length, a corrected value of $K$ is found for each line. $K$ is equal to the reciprocal of the slope since from the equations

$$
\text { and } \begin{aligned}
\mathrm{KD}_{0}^{2} & =\varepsilon \\
K D_{1}^{2} & =1+\varepsilon \\
K & =\frac{1}{D_{1}^{2}-D_{0}^{2}}=\frac{1}{\text { slope }}
\end{aligned}
$$

Thus from the corrected value of $K$ a corrected value of the slope of each line may be found which is obtained by using all measurements on the plate, rather than those of one line alone. In practice it was found that using the eleven iron standards to obtain the mean plate constant gave as good results as using















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3+1=502 \quad \mathrm{~b} 123
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all the lines measured. This method was used by Lacount ${ }^{7}$ in his interferometer measurements, and thus his results have a smaller probable error than those of Gale, Monk and Lee ${ }^{6}$. However, in obtaining the slope of the curve for each line Lacount used only two ring diameters. This is theoretically sound since a straight line may be determined by two points. But if more points are plotted, certainly greater accuracy should be obtained, particularly in the final step in obtaining $\varepsilon$ in which $D_{n}^{2}$ is divided by the corrected slope. Then too it seems reasonable to use all the data available on the plate. The writer first tried the method used by Lacount of measuring the diameters of two rings only and obtaining the mean slope from that. After measuring three plates (12, I3 and 14) and calculating the wave lengths of the hydrogen lines, spreads as high as .007 Au . were found. This was too high since good accuracy to three decimal places was desired.

On the last three plates (16, 17 and 18) the diameters of four rings were measured on each line. On one plate (18) the calculations were made by the method used by Gale, Monk and Lee ${ }^{6}$. Curves like that in figure 3 were plotted for each line on large cross-section paper ( $50 \times 100 \mathrm{~cm}$.$) and \varepsilon$ found for each line by stretching a thread so that it best fitted the points and reading off $\varepsilon$. On the whole the wave lengths checked fairly well with the averages of those already found. A few were considerably off, and the writer found that by restretching















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the thread through the plotted points he could invariably
obtain a new value of $\varepsilon$ that would give better results: This method was discarded as not being accurate enough. Besides, it proved to be physically tiring to plot the points and stretch the thread on such large cross-section paper.

$$
\text { For plates } 16 \text { and } 17 \text { three values of the slope were }
$$ obtained for each iron line:

$$
\frac{D_{3}^{2}-D_{0}^{2}}{3}, \quad \frac{D_{3}^{2}-D_{1}^{2}}{2} \text { and } \quad \frac{D_{2}^{2}-D_{0}^{2}}{2}
$$

These were averaged, a plate constant obtained for each line, and a mean plate constant obtained from the eleven iron lines. The hydrogen wave lengths calculated by this method for these plates checked very well, but were lower than the averages obtained from the first three plates. Then for plates 16 and 17 a mean plate constant was obtained by using all 36 lines (11 iron and 25 hydrogen). This plate constant was higher for each plate than the one obtained from the iron lines alone. This gave a larger value for $2 t$ for each plate but, surprisingly enough, the calculated values of the hydrogen wave lengths came out practically the same. This is explained as follows:

$$
\lambda=\frac{2 t}{p} \text { and } 2 t=p_{s} \lambda_{s}
$$

where $p_{s}$ is the order of interference of standard wave length $\lambda_{s}$

$$
\text { Therefore } \quad \lambda=\frac{p_{S} \lambda_{S}}{p}
$$





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 $\frac{A D I}{q}=x \quad \operatorname{arghotarat}$


Figure 4


If the plate constant is increased, $\varepsilon_{s}$ is increased and $p_{s}$ is increased. But if the same plate constant is used for the hydrogen lines, $p$ is increased by nearly the same percent as $p_{S}$ and hence $\lambda$ remains about the same. Both numerator and denominator of the fraction $\frac{2 t}{p}$ have been increased. This brought out the fact that great accuracy in the slope of a line is not so essential provided a corrected slope is obtained for each line from the mean plate constant so that the slope will have the same percentage error for all the lines.

But if the results are to be accurate to three decimal places, the $\varepsilon$ obtained for each hydrogen line must be accurate. Now

$$
\varepsilon=\frac{D_{n}^{2}}{\text { slope }}-n
$$

If the value substituted for the slope in this equation is correct and all the values of $D_{n}^{2}$ are correct it makes no difference which value of $D_{n}^{2}$ we use. But if the slope is in error the value obtained for $\varepsilon$ will depend upon the value of $D_{n}^{2}$ employed. For example, suppose curve 1 in figure 4 represents the actual slope and assume that the "corrected" value of the slope as obtained above is too small. Then if we substitute $D_{0}^{2}$ in the above equation we should get the value $\varepsilon$, for $\varepsilon$. If we substitute $D_{2}^{2}$ we should get $\varepsilon_{2}$. Curves 2 and 3 are drawn having the "corrected" slope. It would seem then that $D_{0}^{2}$ should be used in every case since a line drawn through it with a slope slightly in error will give a value for $\varepsilon$ more nearly























equal to the accurate value. However, we have seen that errors in $\varepsilon$ tend to cancel out if the same error occurs in the $\varepsilon$ 's of the iron lines (which are used to get 2t) as occurs in the $\varepsilon$ of a hydrogen line (which is used to get p). Hence it would be best to use the corrected value of $D_{n}^{2}$ which can be calculated with the greatest accuracy.

It would not be logical to use simply the square of a measured diameter. This would have a high probable error since it depends on the settings made on only one ring. We should calculate a corrected value of $D_{n}^{2}$ which depends on all the settings made on all the rings of one line. Although the accuracy of the slope does not now seem so important it would be convenient to calculate it by the same method that is used in obtaining a corrected value of $D_{n}^{2}$. Of course, both these could be obtained by plotting a curve on cross-section paper but the writer feels that for himself at least this method would not be dependable.

There comes to mind the method of least squares as probably the best method of finding the slope of a line drawn through 4 or 5 experimentally located points, and also of finding corrected values of the points.

Referring to figure 3

$$
\text { Let } \begin{aligned}
y_{n} & =D_{n}^{2} \\
a & =\text { slope of line } \\
b & =y \text { intercept }
\end{aligned}
$$























$$
0=
$$

$\operatorname{ant5} 520-290+2=$

$$
\text { gyinetejnt }=4
$$

$$
\begin{align*}
& \text { Then } y_{n}=a n+b \\
& \text { Using the least squares method } \\
& \left(y_{n}-a n-b\right)^{2}=\nabla_{n}^{2} \\
& v_{0}^{2}+v_{1}^{2}+v_{2}^{2}+\cdots \cdots \cdot=\Sigma\left(v_{n}^{2}\right)=M \\
& \frac{\partial \mathbb{M}}{\partial a}=0=\sum_{n} 2\left(y_{n}-a n-b\right)(-n) \\
& \left(-0 y_{0}-l y_{1}-2 y_{2} \cdots \cdot n y_{n}\right)+\left(0 a+1^{2} a+2^{2} a+\right. \\
& \left.\cdots \cdots \cdot n^{2} a\right)+(0 b+1 b+\cdots \cdot n b)=0 \\
& \frac{\partial M}{\partial b}=0=\sum_{n} 2\left(y_{n}-a n-b\right)(-1) \\
& \left(-y_{0}-y_{1}-y_{2}-\cdots \cdot y_{n}\right)+(0 a+1 a+2 a+\cdots n a)+(n+1) b=0 \\
& \text { Let } A=D_{I}^{2}+2 D_{2}^{2}+3 D_{3}^{2}+\cdots \cdot n D_{n}^{2} \\
& B=I^{2}+2^{2}+\cdots \cdot \cdot n^{2} \\
& C=1+2+\cdots \cdots \cdot n \\
& F=D_{0}^{2}+D_{1}^{2}+\cdots \cdots D_{n}^{2} \\
& n=\text { number of } \operatorname{ring}(0,1,2, \cdots n) \\
& N=\text { total number of rings }=n+1 \\
& \text { Then } \mathrm{Ba}+\mathrm{Cb}=\mathrm{A} \\
& \mathrm{Ca}+\mathrm{Nb}=\mathrm{F} \\
& \text { and } a=\frac{A N-C F}{B N-C^{2}}  \tag{5}\\
& b=\frac{B F-A C}{B N-C^{2}} \tag{6}
\end{align*}
$$



$$
\begin{align*}
& D_{n}^{2}=b+n a  \tag{7}\\
& \frac{D_{n}^{2}}{a}=n+\varepsilon \tag{8}
\end{align*}
$$

A value of the slope (a) may be calculated for each iron line by means of equation 5. The plate constant for each line $\left(\frac{\lambda}{a}\right)$ may be calculated and the mean plate constant obtained. From this a corrected value of the slope for each line may be obtained.

Next comes the problem as to which corrected value of $D_{n}^{2}$ can be calculated with the smallest probable error. To solve this problem let us use the "cut and try" method.

Let the correct values of $D_{n}^{2}$ for five rings equal 1,3 , 5, 7, 9

Let the correct slope $=2$
Now suppose that $D_{0}^{2}$ is in error by +.01 and calculate by the least squares method corrected values of $D_{n}^{2}$

$$
\begin{array}{lcl}
D_{n}^{2} & n D_{n}^{2} & C=0+1+2+3+4=10 \\
D_{0}^{2}=1.01 & 0 & B=0+1^{2}+2^{2}+3^{2}+4^{2}=30 \\
D_{1}^{2}=3 & 3 & B F=750.3 \quad A N=350 \\
D_{2}^{2}=5 & 10 & A C=\frac{700}{50.3} \\
D_{3}^{2}=7 & 21 & C F=250.1 \\
D_{4}^{2}=9 & \underline{36} & B N-C^{2}=50 \\
F=25.01 & A=70 &
\end{array}
$$

(8)
(a)

$$
3+\pi=\frac{a^{6}}{n}
$$


 a
 - bomi itano







$$
\mathrm{c}_{1}=\mathrm{s}+8+8+\sin 0=
$$

$$
0 \varepsilon=
$$

$$
=8 D+\dot{i}+s^{3}-1+1+0=
$$

GEE
$\# 1 \cdot=$
$\qquad$ $=$ $\qquad$
${ }_{0}^{8}$
L.

$$
=O A
$$

$$
\begin{aligned}
& 5 \\
& =-3-18
\end{aligned}
$$





$$
\pm 4
$$

$$
08=
$$

$$
\begin{aligned}
& b=\frac{B F-A C}{B N-C^{2}}=\frac{50.3}{50}=1.006 \\
& a=\frac{A N-C F}{B N-C^{2}}=\frac{250.1}{99.9}=1.998
\end{aligned}
$$

From these vaules of $a$ and $b$ we $c a n$ obtain, by means of equation 7 , the corrected values of $D_{n}^{2}$ given in the first column of table 1. The other four columns are obtained by assuming an error of +.01 for $D_{1}^{2}, D_{2}^{2}, D_{3}^{2}$ and $D_{4}^{2}$.

## Table 1

ERRORS IN $D_{n}^{2}$ DUE TO ERRORS IN MEASURTMENT

| Error of +.01 in |  | $D_{0}^{2}$ | $D_{1}^{2}$ | $D_{2}^{2}$ | $D_{3}^{2}$ | $D_{4}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Corrected value of | $D_{0}^{2}$ | 1.006 | 1.004 | 1.002 | 1.000 | .998 |
|  | $D_{1}^{2}$ | 3.004 | 3.003 | 3.002 | 3.001 | 3.000 |
|  | $D_{2}^{2}$ | 5.002 | 5.002 | 5.002 | 5.002 | 5.002 |
|  | $D_{3}^{2}$ | 7.000 | 7.001 | 7.002 | 7.003 | 7.004 |
|  | $D_{4}^{2}$ | 8.998 | 9.000 | 9.002 | 9.004 | 9.006 |

Examination of table 1 shows that the error in $D_{2}^{2}$ is always $\frac{1}{5}$ the error in any one line while the error in $D_{0}^{2}$ or $D_{1}^{2}$ is larger if $D_{0}^{2}$ or $D_{l}^{2}$ is in error. Now since the two inner rings are broader than the rest it is more difficult to measure their dianeters accurately. Thus the probable errors in the measured values of $D_{0}^{2}$ and $D_{1}^{2}$ would be larger than for $D_{2}^{2}$. Hence the probable errors in the corrected values would be

## Table 2 <br> Arrangement of Data




## Table 3

## Calculation of $2 t$

Iron Standards
Plate 13


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larger. In the case of $D_{2}^{2}$ the error would be $\frac{1}{5}$ the algebraic sum of the errors of all the rings if there are five rings measured. Beyond the third ring on many lines the rings are faint and difficult to measure. Thus it seems that a corrected value of $D_{2}^{2}$ would be the best to use in obtaining, the $\mathcal{E}$ for each line. $\varepsilon$ may be calculated from equation $8 \frac{D_{2}^{2}}{a}=2+\varepsilon$

Plates 16,17 , and 18 were recalculated using this method. The work involved in calculating $a$ and $b$ by this method was not as tedious as might appear. Since four rings ( $\mathbb{N}=4$ ) were measured in each case $B N-C^{2}$ was the same for all lines

$$
\begin{aligned}
& B=I^{2}+2^{2}+3^{2}=14 \\
& C=1+2+3=6 \\
& B N-C^{2}=20
\end{aligned}
$$

The results obtained on these three plates by this method were in such good agreement that plates 12,13 and 14 were remeasured so that this method could also be used with them.

Table 2 shows how the data were arranged and part of the calculations made for one iron line of plate 13.

Table 3 shows how the remainder of the calculations for obtaining $2 t$ were recorded.

E for each hydrogen line was calculated just as for each iron line, using a corrected value of $D_{2}^{2}$ and the corrected slope obtained frorn the iron standards. Table 4 shows the results obtained from plate 13 .
(

$$
\text { Table } 4
$$

Hydrogen - Tabulation of Results - Plate 13


Since the exact values of the wave lengths of the hydrogen lines are unknown the only way to judge the various methods of calculating them seems to be by comparing results obtained from several plates. If they check with small deviations and small probable errors the results must be considered good. The writer found that the final method of least squares as described above gave the smallest deviations and the smallest probable errors of all the methods tried.

## RESULTS

The results obtained from all six plates are shown in table 5. The intensity of each line as listed is the mean intensity for all six plates. A line of average intensity is indicated by $0,0_{4}$ indicating the weakest and 5 the strongest lines on the plate.

The spread, average deviation and probable error are shown in the last three columns. The writer feels that the probable error as calculated in the usual manner does not mean much where there are only six determinations to average. The actual probability of error must be arrived at by a consideration of the spread and the deviation from the mean. The maximum spread was . 0032 Au. and the mean spread was . 0021 Au. About one half of the mean spread would seem to be a conservative estimate of the actual probability of error. The writer will state that he feels his values are correct to within $\pm .001 \stackrel{\circ}{\mathrm{~A}}$. This is Huch larser than the calculated probable error which is .0002 Au
(

## Table 5 <br> Correlation of Results


but other factors must be taken into consideration. One is the possibility of systematic errors. Something in the set up of the apparatus may make all the values high or low. Also the Writer feels that there is some doubt as to the accuracy of some of the iron standards recommended by the International Astronomical Union ${ }^{4}$. For certain iron standards the calculated values of $2 t$ were always lower than the average. This would seem to cast some doubt on the correctness of those standards. For example, if four iron standards which consistently gave low values for $2 t$ were omitted and the calculation based on the other seven iron standards alone, the results would be increased by about $.00002 \%$. This would mean an increase of .001 Au. for each wave length.

Thus when all the sources of error are considered the probable error of $\pm .001 \mathrm{~A}$. seems reasonable and as small as can be claimed. In spite of this rather large probable error the writer feels that these results are good when compared with any previous ones that he has seen.

In conclusion the writer wishes to express his thanks to all who have assisted him in this work. In particular he wishes to thank Professor Norton A. Kent who directed the work, Professor Royal M. Frye who made many helpful suggestions including the least squares method of calculation, Dr. Reginald G. Lacount who assisted in the arrangement and alignment of the apparatus and Mr. Royal C. Bryant who assisted in taking the


## ABSTRACT

A Fabry-Perot interferometer (etalon) crossed with a plane reflection grating in a Littrow mounting was used to obtain six interferometer plates of the molecular spectrum of hydrogen between $\lambda=4900$ Au. and $\lambda=5900$ Au. A standard Pfund arc was used to obtain secondary iron standards on each plate.

Eleven lines in the iron spectrum recommended as stendards by the International Astronomical Union ${ }^{2}$ and twenty-five hydrogen lines were selected to be measured. The diameters of the interference rings of those lines were measured on a Gaertner comparator, four or five rings being measured on each line.

Several methods of calculating wave lengths were tried. and a least squares method gave the best results. With this method the following relations, developed from the theory of the etalon were used:

$$
\begin{align*}
& p \lambda=2 t  \tag{I}\\
& p=p+\varepsilon  \tag{2}\\
& n+\varepsilon=K D_{n}^{2} \tag{3}
\end{align*}
$$

$K \lambda=$ the plate constant
where $t=$ distance between reflecting surfaces of the etalon $\lambda=$ wave length

1. N.A.Kent and R.G.Lacount A Spring Suspended Thermostated Littrow Spectrograph. Jour. Opt. Soc. Am. 28, 7, 1938
2. Transactions of the International Astronomical Union III, 86,1928

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p = order of interference at the center of the ring system
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$P=$ integral order of interference
$\varepsilon=$ fractional order of interference
$n=$ number of the $\operatorname{ring}(0,1,2,3$, etc)
$D_{n}=$ diameter of the $n t h$ ring
$K=$ constant for a single wave length
Referring to equation 3 , if $D_{n}^{2}$ be plotted against $n$ for any wave length a straight line is obtained as in figure 1 whose slope equals $\frac{1}{\mathrm{~K}}$. Then if $D_{n}^{2}$ be multiplied by the reciprocal of the slope, $\varepsilon$ may be obtained.

The slope of this line for each iron standard was obtained by the method of least squares, and $K \boldsymbol{\lambda}$ calculated. The mean value of $K \boldsymbol{\lambda}$ for the eleven iron standards was used as the plate constant.

A corrected slope for each iron standard was obtained by dividing the plate constant by $\lambda$. Then $\varepsilon$ was obtained by multiplying $D_{2}^{2}$ by $\frac{1}{\text { slope }}(K)$. This value of $D_{2}^{2}$ was a corrected value obtained by least squares calculation and not the square of the measured value of $D_{2}$. $D_{2}^{2}$ was used because it was considered that the probable error in calculating it was less than for any other value of $D_{n}^{2}$. $P$ was obtained for each iron standard by dividing an approximate value of $2 t$ by $\lambda$. Then $p=P+\varepsilon$ and a corrected value of $2 t$ was obtained by using equation 1. The mean value of $2 t$ obtained for the eleven iron
||cccele


Figure 1
Graph of $K D_{n}^{2}=n+E-D_{n}^{2}$ plotted against $n$

standards was calculated. This was the value of $2 t$ used later in calculating the corrected wave lengths of the hydrogen lines.

A corrected slope was obtained for each hydrogen line by dividing the plate constant by an approximate value of $\lambda$. These approximate wave lengths were obtained from the measurements of Gale, Monk and Lee ${ }^{3}$. $\varepsilon$ for each hydrogen line was obtained as for the iron standards. Then $P=\frac{2 t}{\lambda \text { (approx) }}$ $p=P+\varepsilon$ and a corrected value of $\lambda$ was obtained from equation 1.

The results obtained on all six plates are shown in table 1. A line of average intensity is indicated by $0, \mathrm{O}_{4}$ indicating the weakest and 5 the strongest lines on the plate.

The spread, average deviation and probable error are shovn in the table. The writer feels that the probable error as calculated in the usual manner does not mean very much where there are only six determinations to average. The maximum spread was $.0032 \AA$ Au. and the mean spread was . $0021 \AA$ Au. About one half of the mean spread would seem to be a conservative estimate of the actual probability of error. The writer feels that his values are correct to $\pm .001 \AA$. This is much larger than the calculated mean probable error which is .0002 Au. but other factors must be considered. For example there seems to be some doubt as to the accuracy of some of the iron standards recommended by
||cccelen

## Table 1

Correlation of Results


Mean $2.1 .6 \quad .2$

the International Astronomical Union ${ }^{2}$. For certain iron standards the calculated value of $2 t$ was always lower than the average. If four iron standards which consistently gave low values for $2 t$ were omitted and the calculations based on the other standards alone, the results would be increased by about $.00002 \%$. This would mean an increase of .001 Au . for each wave length. Hence the probable error of $\pm .001$ seems reasonable.


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## AUTOBIOGRAPHY



The writer was born in Seabrook N. H. August 22, 1896. His father was Stephen E. Combes and his mother, Julia Putney Combes. They moved to Amesbury, Mass. where the writer attended the Amesbury High School graduating in 1914. He then attended Mount Hermon School graduating in 1916. He was then commissioned 2nd Lieutenant in Field Artillery in 1917. He received a B.S. degree from Wesleyan University in 1921 and an M.A. degree from Boston University in 1928. He taught science in the Medford and Pittsfield High Schools in 1921 and 22. He was Instructor in Physics at Simmons College from 1922-1925. He taught Physics in the Haverhill High School in 1925-1926. He was Instructor of Physics in Tufts College from 1926-1928
and was appointed Assistant Professor of Physics in 1928. He was married in 1926 to Viola I. Grethe (B. U. 1922).

They have two children, Frederick Lewis, born in 1929 and Everett Tapley born in 1935.

